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Developing Hypothetical Learning Trajectory for Green's Theorem

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Abstract

This research is aimed at developing Hypothetical Learning Trajectory (HLT) for Green Theorem course material that deals with double integral and line integral. The theorem is very useful for students of Mathematics and Engineering. This research applies educational research method developed by Gravemeijer and Cobb with the following stages: 1). Experiment preparation, 2). Experiment design, and 3). Retrospective analysis. This research involved 24 students taking Multivariable Calculus class as the research subjects. The research found that the Hypothetical Learning Trajectory has met the criteria of validity and practicality for it has been through one-to-one trials and field tests, and all instructionals being designed thereto can run as expected

Keywords: HLT, Green Theorem, Educational Research Design, Validity, Practicality

1.Introduction

The Mathematics application given in the first year students is Calculus. There are many applications in Calculus for various fields Mathematics, Engineering, Economics, Biology, Medicine and others. One piece of high-level material from Calculus is the use of the Green's Theorem. In the field of Engineering, Economics and Physics, the subject matters of Mathematics which must be mastered by the graduates shall be Calculus concept and its Green's theorem. Green's theorem is part of Calculus which requires the support of Differential and Integral Calculus. Green's theorem expresses a relationship between the line integral around a simple closed curve C and the double integral over the extent in area D bounded by C. This theorem derives from the name of George Green and constitutes a special two-dimensional case of the more general theorem, namely Stokes' Theorem. Green's theorem was discovered when Vector Calculus is discussed and it requires an understanding to the initial capabilities of Calculus which is the main requirement for mastering this theorem.

As the lecturers of Vector Calculus, we found several obstacles in giving lecture about Green's Theorem, mostly because: 1). The students hardly understood the basic concepts of differentiation and integration techniques, 2). The students often mistakenly translated the areas of integration in a double integral, something which could actually be solved through line integrals. In general, the concept of Line Integral is a generalization of Ordinary Integrals. Besides, the learning methodology being adopted so far is still conventional in nature, in which the learning material and assignments are just given and instructed traditionally at each meeting without guiding the students to follow good learning trajectory.
Based on the above-stated background and in order to improve the capability of the students to prove the concept of Green's Theorem, this paper discusses how to design HLT for Green's Theorem material.

2.Literature Review

One of the efforts to create quality learning which always experiences progress to positive direction is by means of preparing didactic design, namely by designing Hypothetical Learning Trajectory (hereinafter shall be referred to as "HLT") or also called Learning Trajectory, as suggested by Simon (1995). The word "hypothetic" indicates that part of the learning trajectory is always flexible. According to Simon (1995), there are three main components of Learning Trajectory, namely 1. Learning Goals, 2. Learning Activity, and 3. Hypothetical Learning Process. Then, Clements & Sarama (2004) developed it to become: 1). Learning Objectives, 2). Progress in the Development of Thinking and Learning, and 3). Stages of Instructional Assignment. Furthermore, Larson et al (2017) added the fourth aspect, namely: 4). The Role of Instructors (Lecturers) in Supporting Mathematical Development of the Student at the Stages of Assignment. Simon & Tzur (2004) presented four principles that must be considered in developing HLT;(a) HLT should be developed based on the existing level of understanding of the students,(b)HLT is a vehicle for planning certain mathematical concepts in a learning process, (c)Instructional assignments or worksheets as the tools for improving the understanding in mathematical contents are the keys to the learning process, (d)Instructor /Lecturer must be able to modify every aspect of HLT from time to time because of hypothetical and inherently uncertain nature of this process.

Simon and Tzur (2004) described the mechanism for developing mathematical concepts based on constructivism aspect, namely the assimilation construct based on Piaget's theory. Piaget asserted that students cannot simply accept new concepts without building new concepts through assimilation and accommodation of the previous concepts. HLT has been used for developing several materials, among others by: Larson et al (2017) who designed LT for Matrix as a linear transformation, Sari & Julianti (2018) who designed LT for understanding the concept of angles, Brophy & Lowe (2017) who made LT to think critically in programming, Swidan (2019) who used LT for material in Calculus Basic Theorem, Meika et al (2019) who inserted LT in material combination, Fauzan & Sari (2017) who used LT for understanding the concept of Fractions, Tamba et al (2018) who designed LT for quadratic disproportion, Bisognin et al (2019) who applied LT for the length of curve, Arnold et al (2018), Syafriandi (2018) and Campos & Fontanari (2019) for Statistics, Nusantara & Putri (2018) for the slope of the line, Clements et al (2019) for the material of geometric shapes, Czarnocha, 2016) for linear equations, and Rahayu & Wijaya (2018) who designed LT for statistical thinking. In this design research, HLT is designed in such a way by following the format proposed by Clements and Sarama (2004) as well as by Larson. et.al (2017).

2.1.The Purposes of Green's Theorem Learning

The final goal of Green's Theorem learning is for the students to be able to prove and analyze the relationship between double integral (ability to calculate the extent of an area) and line integral. The first ability that must be possessed by the students is mastering the concepts of double integral and line integral, and understanding the concepts of Area and Length of a curve.

2.2.The Progress of Thinking and Learning Development

The arrangement of thinking and learning development and the sequence of instructional assignments are made based on current didactic analysis, whether those used by lecturers or the results of research and findings by experts, including the constraints and mathematical knowledge about Green's Theorem. From the analysis of textbooks, RPS and handouts used by lecturers, it was found that the lecturers only focused on analytical evidence with examples. One of the obstacles faced by students in the previous learning is their failure to fully understand of the completion of double integrals and line integrals. In completing the double integral, the students still find it difficult to form any random area to become the first type and second type and express them. While in line integrals, the students find it hard to solve the matters which in particular relate to the way to express the curves as the limit (pathway) of integral.

2.3.The Instructional Stage and the Role of Lecturers

The instructional stage for guiding students' activities was designed in view of the needs for learning materials and the conjectures to the difficulty factors faced by the students. Preparation of this instructional stage had undergone several improvements, because no activities interrelated between one stage to another had yet to be seen. Initially, the role of the lecturers was included in each activity, but because HLT was aimed at developing the concepts independently, such role was created in the form of predictions and conjectures for each progress. For observing the process of running the learning trajectory, the students were assigned to fill in the worksheets in accordance with the instruction given.

3.Methodology

The research method being applied in this research is the Design Research as proposed by Gravemeijer and Cobb (2013) in Nieveen, (2013) which constitutes as part of Plomp's steps. The stages having been went through are preceded by a cyclical process, namely 1) preparation for the experiment (preparation stage), 2) design experiment (experimental design stage), and 3) analysis retrospectively (retrospective analysis stage). This Design Research is also known as Development Research and developed by Plomp theory. According to Plomp, Research Development is an alternative research aimed at designing and developing educational interventions regarding learning process, learning environment, and the like with the purpose of developing or validating theories. This design research is focused on developing the order of presentation of material in mathematics learning. At the initial stage, it is started by conducting a thought experiment, namely thinking about the route or trajectory of the learning (instructional) that will be experienced by the students. At this stage, it is considered to examine the conjectured local instruction theory being used thus far. Furthermore, the results of thought experiment were tested in the classroom in accordance with Plomp's steps. Based on the result of reflections to the results of experiments in the classroom, the thought experiment was continued regularly. Such a cyclic process is as shown in Figure 1.

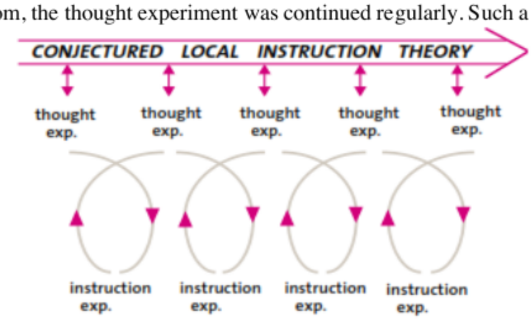

Figure 1. The Process of Design Research According to Gravemeijer and Cobb

4.Results and Findings

4.1. Results at Experiment Preparation Stage

Discussions with the lecturers of Green's Theorem subject revealed that it is a common practice that Green's Theorem was presented in the same traditional way, namely by giving explanation and proving the application of the Theorem slightly, and then showing the examples of their usage. After making evaluation to the understanding of the students to the Theorem it showed that the results of learning in such a way were rather unsatisfactory. Therefore, at this phase there are two measures being taken for finding solution to such condition: First, reviewing any literatures on HLT, and the literatures on how to teach Green's Theorem. Second, designing HLT and its accessories. In designing HLT, the end points were determined from several activities which shows the main goal of each progress namely the goal to be achieved through various activities for proving the Green's Theorem. This goal was set as the guide for learning activities being designed. After that, the starting points were determined, namely the initial knowledge or the informal knowledge already acquired by the students from the definitions for carrying out various activities.

Gravemeijer and Cobbs stated that a design will work if its framework can define what norms must be addressed, and in this case, one of such norms is RME theory which is applied appropriately since it can play dual role; not only for guiding the design, but also offering the framework for interpreting student learning process. This designed HLT incorporates RME elements which are tailored with the capability of the students who have already understood the concept of Area and Perimeter and Work being done by a force. After going through correction for 3 (three) times as validated by the expert from Padang State University, the followings are the descriptions on instructional stages or activities based on the goals and conjectures of thought of the students;

Goal 1: Understanding the relationship between the area of a certain flat shape and its circumference

Activity 1.1: The lecturer asks the students to explore the topic regarding Area and Perimeter of certain flat shape by giving instruction as follows: "Please locate the area of a circle with radius r and its circumference. Can you find any similarity between the two forms by doing a little algebraic manipulation?"

Activity 1.2: The students are instructed to investigate the same problem occurring for other known shapes.

Goal Conjecture 1: Some students may quickly understand the area of a circle and the circumference of a circle and find the relationship that the area of a circle is equal to the circumference of a circle multiplied by ½ of the length of its radius. The students can also find a relationship that the area of a square is equal to circumference multiplied by ¼ of the length of its lateral area.

After the students have gained understanding that there is a relationship between the area of a certain flat shape and its circumference, then it is applied to any other flat shapes as shown by the following activities;

Goal 2: Understanding the relationship between the extent of any area and its surroundings

Activity 2.1: The lecturer asks the students to do the same analysis for any other shapes as shown in Figure 2(a) and (b) by using the integral concept.

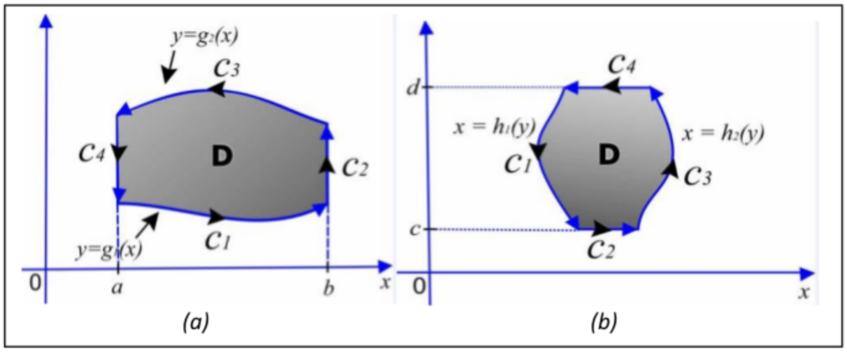

Figure 2. The area of any shapes formed by a curve

The lecturer reviews the definition of double integrals for calculating the extent of an area, with $\iint_D dA$ and analyze the integral $\iint_D \left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dA$ where P and Q is the component of field vector $F(x,y)=(P(x,y),Q(x,y))$ with $F(x,y)=(-y,0)$, $F(x,y)=(0,x)$ and $F(x,y)=(-y/2,x/2)$.

Activity 2.2: The lecturer reviews the definition of line integral which takes the form of $\oint_C \vec{F}\bullet dr$ for the vector functions in activity 1, and assigns the student to analyze how the integral form looks like.

Activity 2.3: The lecturer instructs the students to determine the line integral of vector functions in activity 1 along the line (path) according to Figure 1.(a) and (b).

Goal Conjecture 2: For activity 1 the students can calculate the area by recalling the extent of the area bounded by two curves (for fig 2(a)), which may become a double integral.

$$A(D)=\int_a^b \left(g_2(x)-g_1(x)\right)dx = \int_a^b\int_{g_1(x)}^{g_2(x)} dy\,dx = \iint_D dA \quad ; D=\{(x,y)\mid a\le x\le b, g_1(x)\le y\le g_2(x)\}$$

As for figure (b) the findings shall be as follows;

$$A(D)=\int_c^d \left(h_2(y)-h_1(y)\right)dy = \int_c^d\int_{h_1(y)}^{h_2(y)} dx\,dy = \iint_D dA \quad ; D=\{(x,y)\mid c\le y\le d, h_1(y)\le x\le h_2(y)\}$$

Furthermore, the students found the followings for all three vector functions that $A(D)=\iint_D dA$. Because $\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)=1$.

For activity 2 the students found:

$$\int_C \vec{F}\bullet dr = \oint_C x\,dy \qquad (1)$$

$$\int_C \vec{F}\bullet dr = -\oint_C y\,dx \qquad (2)$$

$$\int_C \vec{F}\bullet dr = \tfrac{1}{2}\oint_C -y\,dx + x\,dy \qquad (3)$$

As for activity 3, the students can prove it by applying the following steps:

For equation (2), by using Figure 1 the students can calculate the extent of the area by using the following formulation:

For proving of equation (2) Figure 1 (a) is used, namely;

$$-\oint_C y\,dy = -\left(\int_{C_1} g_1(x)dx + \int_{-C_3} g_2(x)dx\right) = \int_a^b g_2(x)dx - \int_a^b g_1(x)dx$$

The integrals on paths C2 and C4 are zero with the virtual parameters of x = x and y = f (x).

In the same way it can be proven for equation (1) by taking the parameters y = y and x = h (y) and by taking the area in figure 2. Whereas proof (3) is a combination of (1) and (2).

After the students can prove the relationship between area and the circumference of any shape, they are led to get to the ultimate goal, namely Green's Theorem.

Goal 3: Understanding the relationship between double integrals and line integrals

Activity 3.1: The lecturer instructs the students to analyze the form of line integral and double integral for any vector field $F(x,y)=(P(x,y),Q(x,y))$ and $\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)\neq 1$ and prove it.

Activity 3.2: The lecturer instructs the students to determine an effective way to solve the problem of determining the effort (W) carried out by the force $F=(P,Q)$ along the trajectory C: $r=(x,y)$ with the formula $W=\int_C F\bullet dr$ in which the vector function is $F(x,y)=\left(x^2+e^{\sqrt{x}},2x+\cos(y^2)\right)$ and the path is in the form of parabola $y=x^2$ and parabola $x=y^2$.

Goal Conjecture 3: The students found that $\iint_D\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dA=\int_C P\,dx+Q\,dy$ or in vector it formed $\iint_D(\nabla\times F)\bullet k\,dA=\oint_C F\bullet dr$ which is known as Green's Theorem.

For activity 1, the students can show that:

$$\oint_C P\,dx=\iint_D-\frac{\partial P}{\partial y}\,dA \tag{4}$$

$$\oint_C Q\,dy=\iint_D\frac{\partial Q}{\partial x}\,dA \tag{5}$$

Equation (4) can be proven using area D as shown in figure 1(a). For the right section (4) it can be shown that:

$$\iint_D\frac{\partial P}{\partial y}dydx=\int_a^b\int_{g_1(x)}^{g_2(x)}\frac{\partial P}{\partial y}dy\,dx=\int_a^b\left[P(x,y)\right]_{g_1(x)}^{g_2(x)}dx\quad=\int_a^b\left(P(x,g_2(x)-P(x,g_1(x)\right)dx \tag{6}$$

If done according to the line integral $C=C_1\cup C_2\cup C_3\cup C_4$ and by taking x=x , y=g(x) (depending on the function path), the result is: $\int_C P\,dx=\int_{C_1}P\,dx+\int_{C_2}P\,dx+\int_{C_3}P\,dx+\int_{C_4}P\,dx$

That therefore $\int_C P(x,y)dx=\int_a^b P(x,g_1(x))\,dx+\int_b^a P(x,g_2(x))dx$

$$=\int_a^b P(x,g_1(x))\,dx-\int_a^b P(x,g_2(x))dx\;=-\left(\int_a^b P(x,g_2(x))\,dx-\int_a^b P(x,g_1(x))dx\right) \tag{7}$$

For paths C_2 and C_4 they have 0 values, and if compared between (6) and (7) then equation (4) are proven in the same way for (5) which can also be proven. As for the applications, the students are more likely to use double integral, because it is very difficult to make it integrated by means of line integral because there is no anti derivative for $\int\sin(y^2)dy$, so that to make it integrated, double integral should be applied on $D=\left\{(x,y)|0\leq x\leq 1,x^2\leq y\leq\sqrt{x}\right\}$ Therefore $\iint_D(2-0)\,dA=2\int_0^1\int_{x^2}^{\sqrt{x}}dydx=2\int_0^1(\sqrt{x}-x^2)dx=2/3$

4.2. Results of Design Experiment Phase.

The purpose of conducting design experiment is neither to prove nor to show that the existing design or the existing local instructional theory work properly. Instead, it is aimed at testing and correcting the conjecture of the local instructional theory developed at initial phase, and giving an understanding on how it works. After the HLT was declared valid by the expert, the trial phase was conducted involving the students of Mathematics Education of Bung Hatta University. At the time of the trial, observation was focused on the student's thinking process and the process of how the student achieved something, including the response of the students and whether HLT (in each activity) functioned properly to help students achieve their goals. If all the above work properly, then the procedure of the process will be observed. If there is any failure, it will be tracked and traced in order to know what has happened and what the cause of it. The trial was carried out in two stages only, namely one-to-one test and field test, because of the limited number of students and class schedules that had entered the final round of lectures. For the first stage of the test, namely one-to-one test, 3 (three) students taking vector analysis subject were involved. The students had not yet received the Green's Theorem material and were divided into three categories, namely those with high, medium and low intellectual potential. Initially, the HLT was designed for sole one-to-one test, but due to the student's thinking process which is still low in understanding about the concepts of double integrals and line integrals, the HLT was implemented in 2 (two) sessions of the test with additional application material after proving the Green's Theorem.

The designed conjecture (activities) worked well for the students with high and medium intellectual potential. The results of student's worksheets as shown in Figure 3 are the first one-to-one test, and those in Figure 4 are the second one-to-one test. At the time of proving the line integral became double integral, there was indeed a vacuum situation because the students did not understand the traversed trajectory and the nature of trajectory which have the tendency of changing direction from negative orientation to positive orientation as an integral boundary. The lecturer had to explain again the concept of trajectories on positively oriented line integrals. Thus the stimulant given by the lecturer was to review again the concept of double integrals and line integrals, so that for larger field tests (in group) these weaknesses can be anticipated later.

At the field tests, as conducted in the actual classes, the students were divided into 4 (four) groups with 3 or 4 members per group. The activities were performed as planned, even though the lecturer sometimes had to remind about the concept of trajectory that must be positively oriented to prove the line integral.

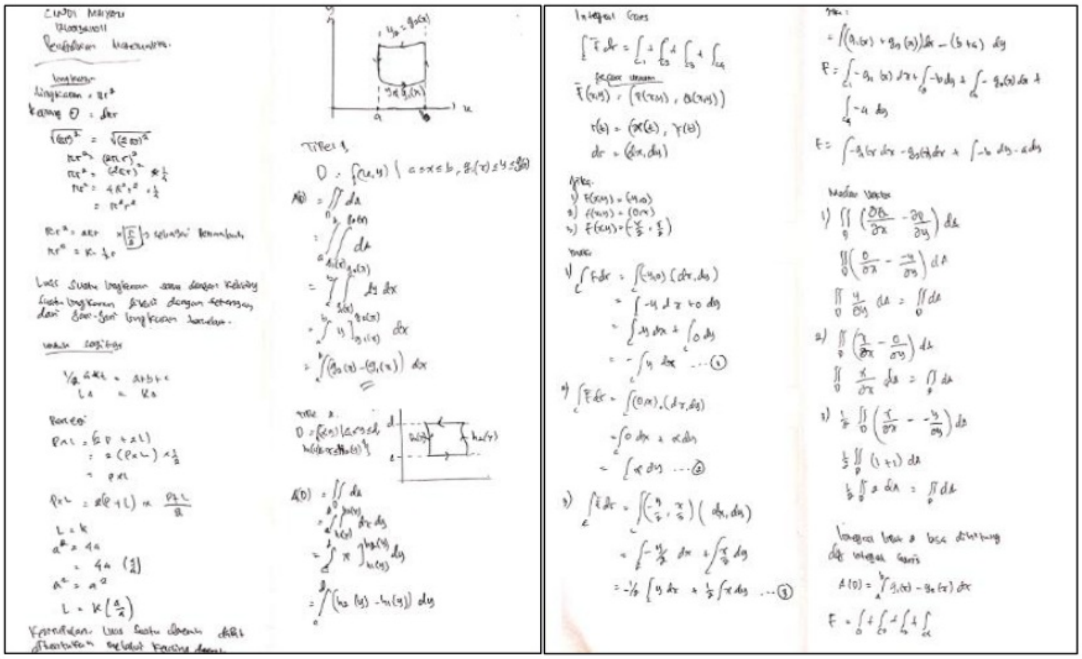

Figure 3. Student's worksheets at the first session

Although there were groups facing problems with the concepts of folding integrals and line integrals, it can be solved by re-explaining the definitions and properties. In the application of the Green's Theorem, in general the students can do it well by using a quick strategy, because it involves double integrals.

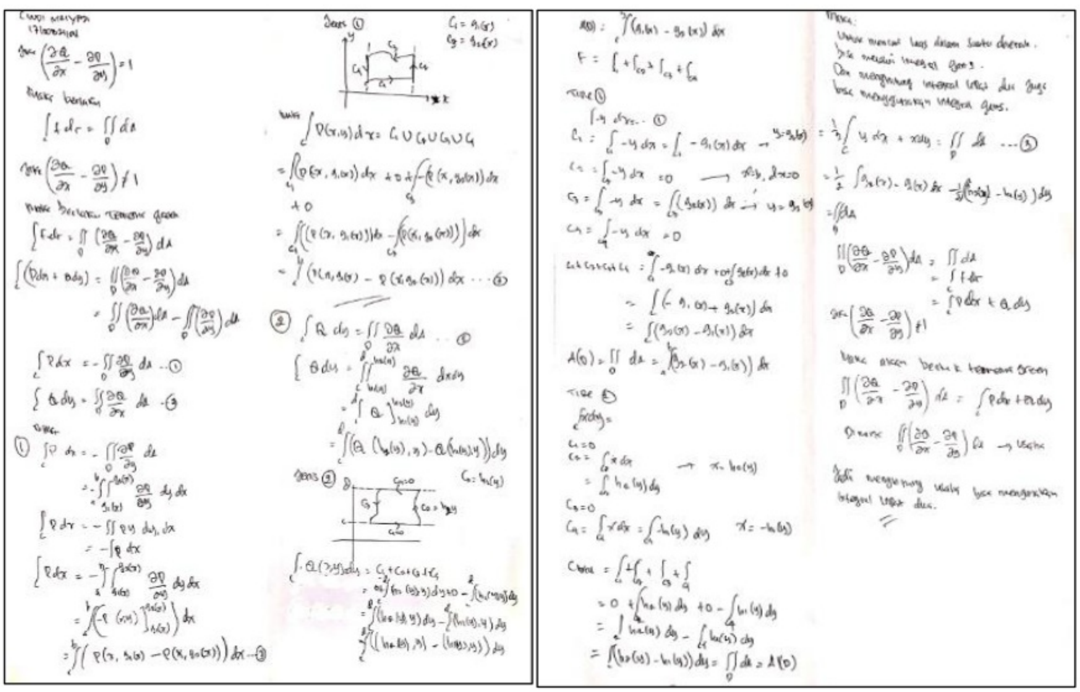

Figure 4. Student's worksheets at the second session

4.3. Results of the Retrospective Analysis Phase.

Retrospective Analysis is highly dependent on the theoretical goal of design experiments, namely to master the way how to prove and apply Green's Theorem in cases relating to double integrals and line integrals. In addition, Retrospective Analysis also contributes to the development of local instructional theory. This analysis is a reflection of the actions taken during the experiments for improving the learning trajectory, especially during one-to-one tests and field tests. One of the findings of the retrospective analysis being conducted by way of interviewing the students shows that some of the students were still hard to understand the concept of integrals and how to solve the integrals. Other than that, they also were not good enough in giving description about the area of integration as a double integral. In view of these results, the lecturers must be able to strengthen the student's understanding on the concept of integral and double integrals as contained in Calculus material. During the field tests, it was found that some of the groups of students were able to follow the activities as expected, but the other groups still had to be guided in such a way by constantly reminding them about the basic concepts of Calculus. However, in general, implementation of the learning trajectory in the classrooms to some extent was a success in accordance with predictions previously designed

5. Conclusions

Learning Trajectory (or HLT / Hypothetical Learning Trajectory) under the topic of Green's Theorem in this paper has met valid criteria because it has complied with the process of designing which involved the experts, and has met the principles and components which should be available in a good HLT. The trajectory being developed has also met the practical criteria because it has passed the one-to-one tests and the field tests and all the instructional being designed can function as predicted. However, this HLT has not yet reached the stage of effectiveness, because it has not yet been tested against the given theories. For further research, the design of this HLT should use technological aspects, such as for concept visualization using Geogebra.

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